NONLINEAR HEURISTIC FOR PUMP OPERATIONS

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ABSTRACT: A nonlinear heuristic is developed for use in obtaining least-cost pump-operation policies for multisource, multitank water-distribution systems. The proposed algorithm links a minimum-cost-constraint identification methodology with a network-simulation model in order to provide the resulting policy. The algorithm has the advantage of being computationally efficient while incorporating the nonlinear characteristics of the water-distribution network. In addition, the algorithm has the added advantage of providing several feasible solutions to the control problem, which then provides the system operator with increased flexibility in selecting a particular policy.

INTRODUCTION

In recent years, the water-utility industry has begun to investigate the use and integration of on-line computers and control technology in improving the daily operations of water-distribution systems. This use has been motivated by a desire to reduce operational costs and provide more reliable operations. One of the greatest potential areas for cost savings is in the scheduling of daily pump operations.

Any real-time control system for use with a water-distribution network will typically contain both an operational SCADA (supervisory control and data acquisition) system and the supporting operational control software. In the current study, the operational control problem will involve determining the optimal pump operation policy for each pump station associated with a specific water-distribution system. The operation policy for a pump station may be defined as a set of rules or guidelines that indicate when a particular pump or group of pumps should be turned on and off over a specified period of time (typically 24 hr). The optimal pump policy is defined as that schedule of pump operations that will result in the lowest total operating cost for a given set of boundary conditions and system constraints. This paper presents a new heuristic based algorithm for use in obtaining the optimal pump policy for a multitank, multisource water-distribution system.

PREVIOUS WORK

Previous attempts at the development of control algorithms for use in obtaining optimal pump policies have generally focused on the use of an implicit formulation in which the optimal control problem is formulated in terms of an implicit state variable such as tank water level or pump-station discharge (Ormsbee and Lansey 1994). Once the optimal time-dependent decision trajectory is obtained, the associated explicit control variables (pump operating times) are then obtained through translation of the implicit solution into an explicit set of decision variables. The implicit formulation may be solved using either dynamic programming (DP) or nonlinear programming depending upon the nature of the implicit state variable and the size of the problem.

Because of the nature of dynamic programming, site-specific DP-based control algorithms have generally not been

readily transferable to other systems. In addition, most applications have been restricted to single reservoir systems or systems with multiple reservoirs in series configurations (DeMoyer and Horowitz 1975; Sterling and Coulbeck 1975a,b; Ormsbee et al. 1989; and Zessler and Shamir 1989). Application of dynamic programming to more complex systems has required the use of complex decomposition strategies (Joalland and Cohen 1980; Carpentier and Cohen 1984; Coulbeck et al. 1988a,b). As a result, the transferability of these algorithms has been limited.

Perhaps the most robust approach for solving the implicit decision variable formulation (at least for multisource, multitank systems) in by using pump-station discharge as the decision variable (Fallside and Perry 1985; Coulbeck and Sterling 1978; Lansey and Zhong 1990). The resulting nonlinear formulation can then be solved using some form of nonlinear optimization in which the system constraints are handled either through the use of penalty terms or an augmented Lagrangian. Despite the improved flexibility afforded by such formulations, assurance of global optimality is normally forfeited and the resulting optimal pump station discharges must be translated into their associated pump policies (Ormsbee and Lansey 1994).

The principal alternative to the implicit formulation is to formulate the control problem directly in terms of the explicit decision variables (i.e., the individual pump operating times). Both Jowitt et al. (1988) and Little and McCrodden (1988) developed control formulations in terms of pump operating times, which were then solved using linear programming. In each case, several simplifying assumptions were made in order to accommodate the nonlinear network hydraulics. In an attempt to avoid such simplifications, Chase and Ormsbee (1989) developed an explicit formulation that was then solved by linking a nonlinear optimization algorithm with a nonlinear network solver (Wood 1980). More recently, Brion and Mays (1991) improved the efficiency of the method by use of an analytically differentiable augmented Lagrangian objective function. Later, Chase and Ormsbee (1991) improved their original formulation by consideration of variable bounds on the individual time intervals. Despite these improvements, the efficiency of the algorithm is highly dependent upon the number of pumps being considered along with the level of discretization of the total operating period (Ormsbee et al. 1992).

PROBLEM FORMULATION

In the proposed approach, the optimal control problem is formulated mathematically as a nonlinear optimization problem. The objective function and associated constraints may be expressed in (1)-(11).

Objective Function

The objective of the optimal pump operation problem is to minimize the energy cost while satisfying the hydraulic

operational requirements of the system. For most water util-ities, the total pumping cost is composed of an energy-con-sumption charge and a demand charge. The energy-con-sumption charge is the portion of an electric bill based on the kilowatthours (kW·h) of electric energy consumed during a billing period. The demand charge represents the cost of hav-ing sufficient facilities on hand to meet peak energy demand, and it is usually assessed on the basis of the peak energy consumption that occurs during a specific time interval (e.g. 15 min). Because of the time variability and nature of the demand charge, the proposed formulation has been restricted to a consideration of the energy charge only. One possible way to incorporate the impact of a demand charge would be to generate a composite solution by repeated applications of the proposed model using a variable demand-charge con-straint as proposed by Jowitt and Germanopoulus (1992). However, in the present paper this extension has been left for future work.

In a typical water-distribution system, the energy-con-sumption cost incurred by the pumping facility depends on the rate at which water is pumped, the associated pump head, the duration of pumping, and the unit cost of electricity. Mathematically the objective function may be expressed as

$$\text{Minimize } Z = \sum_{t=1}^{T} \sum_{i=1}^{I} \frac{0.746\gamma Q_{it} H_{it}}{550 e_{it}} X_{it} r_t \tag{1}$$

where Z = total energy cost to be minimized (\$); T = number of time intervals which constitute the operating horizon; I = number of pumps in the system; γ = specific weight of the fluid (lb/cu ft); Q_{it} = average flow rate associated with pump i during time t [cu ft/sec (cfs)]; H_{it} = average head associated with pump i during time t (ft); X_{it} = duration of time pump i is operating during interval t; r_t = electric rate during time t (\$/kW·h); and e_{it} = average wire to water efficiency asso-ciated with pump i during time t.

For a given network configuration and an associated set of initial and boundary conditions (the vector of initial tank levels **E** and the vector of system demand loadings **M**), the average discharge Q_i, pump head H_i, and pump efficiency e_i, associated with a particular pump i can be expressed as a function of the set of pumps that are operating during the same time period. Since the set of pumps operating during a particular period is explicitly defined by the duration of time a pump is operating (i.e. if $X_{it} = 0$, then the pump is not operating during time period t and if $X_{it} > 0$ then the pump is in operation during period t) the pump discharge and pump head for a particular pump may be expressed as implicit func-tions of the vector of total pump durations for all pumps for a particular time interval. The later condition arises as a result of the fact that the pump head and associated discharge for one pump can be influenced by the pump heads and thus the pump operating times of other pumps. As a result, the ob-jective function may be expressed in terms of the vector of the individual pump operating times as follows:

$$\text{Min } Z \sum_{t=1}^{T} \sum_{i=1}^{I} f[Q_{it}(\mathbf{X}_t,\mathbf{M}_t,\mathbf{E}_t),H_{it}(\mathbf{X}_t,\mathbf{M}_t,\mathbf{E}_t),\ e_{it}(\mathbf{X}_t,\mathbf{M}_t,\mathbf{E}_t),X_{it},r_t] \tag{2}$$

Evaluation of (2) requires that the normal operating ho-rizon (typically 24 hr) be divided into T separate time intervals and that the pump operating time for each pump in each time interval be determined. In the proposed formulation, the time intervals may be constant (e.g. 1 to 12 hr) or they may be varied over the course of the time horizon in order to reflect a time-varying rate structure. For distribution systems with multiple pump stations, with each pump station containing numerous pumps, such a formulation can result in an exces-sive number of decision variables. One way to significantly reduce the total number of decision variables would be to develop a single decision variable for each pump station for each time interval that relates the particular set of pumps in operation during that period. Such a formulation can be ob-tained by rank ordering the various available pump combi-nations associated with each pump station on the basis of unit cost as derived as a function of the existing hydraulic bound-ary conditions (Zessler and Shamir 1989). By rank ordering the pump combinations, the number of decision variables associated with a particular pump station can be reduced from $T \cdot P$ (T = total number of time intervals and P = total number of pumps) to T. A single continuous decision variable can then be developed for each pump station s and each time interval t of the form $X_{st} = II.CC$, where II = is an integer that corresponds to the identification number of the pump combination that operates CC percent (decimal) of the time interval with the understanding that combination $II - 1$ op-erates the remaining $(1 - CC)$ percent of the time interval. (The combination in which $II = 0$ corresponds to the null combination or the decision to run no pumps). For example, a value of $X_{13} = 5.7$ would correspond to a decision to run pump combination 5 (associated with pump station 1) 70% of time interval 3 and pump combination 4 the remaining 30% of time interval 3. Although in theory the variable II can range from 0 to 2^P (where P = total number of pumps in a particular pump station) the size of the set of pump combinations to be considered can be significantly decreased by judiciously prescreening the combinations in order to eliminate inefficient or impractical combinations. As a result, the formulation can be structured so as to only consider those combinations that the operator would like to explicitly con-sider.

It should be recognized that the validity of the rank or-dering formulation will be highly dependent upon the accu-racy of the unit-cost rankings. Since these rankings may change in response to changes in the system boundary conditions, the proposed algorithm evaluates the validity of the current unit-cost rankings at the beginning of each time interval. In the event that the rankings change, the algorithm makes an adjustment to the ranking in order to ensure the validity of the resulting solution.

Modification of the original objective function to accom-modate the proposed rank-ordering formulation yields the following objective function:

$$\text{Min } \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{I_s} f[Q_{it}(X_{st},\mathbf{M}_t,\mathbf{E}_t),H_{it}(X_{st},\mathbf{M}_t,\mathbf{E}_t),$$
$$\cdot e_{it}(X_{st},\mathbf{M}_t,\mathbf{E}_t),X_{it}(X_{st}),r_t] \tag{3}$$

where I_s = number of pumps in pump station s; and S = number of pump stations.

It should be noted that the proposed formulation is very similar to one first proposed by Zessler and Shamir (1989). However, in the current formulation the problem is cast ex-plicitly in term of the pump combination operating times, whereas Zessler and Shamir used pump-station discharges as the decision variables in a dynamic-programming formula-tion. In addition, the formulation of Zessler and Shamir uses a disaggregated approach for handling multiple tank, multiple pump-station systems, whereas the proposed formulation can solve for the decision variables for all pump stations simul-taneously.

CONSTRAINTS

The objective function as expressed in (3) is subject to three different kinds of constraints: (1) A set of implicit system

constraints; (2) a set of implicit bound constraints; and (3) a set of explicit decision variable constraints.

Implicit System Constraints

Evaluation of both Q_{it} and H_{it} as well as e_{it} in (3) requires the development of functional relationships relating each of the parameters to X_{st}, $\mathbf{M}$, and $\mathbf{E}$. Such relationships can be developed implicitly using nonlinear regression analysis of actual or simulated network realizations or explicitly through the use of a network simulation program. (In the current study the latter approach is used.) Application of a network simulation program requires the simultaneous solution of both conservation-of-mass and conservation-of-energy relationships. The nodal conservation of mass constraint $FM(Q)$ requires that the sum of the flows into or out of any junction node j minus any external demand M_j must be equal to zero. This can be expressed as

$$FM(Q)_j = \sum_{n \subset [j]}^{N_j} Q_n - M_j = 0 \text{ for all } j \qquad (4)$$

where N_j = number of pipes connected to junction node j; and $[j]$ = set of pipes connected to junction node j.

The conservation of energy constraint $FE(Q)$ requires that the sum of the line losses (HL_n) and the minor losses over any distinct pipe path or loop k (HM_n), minus any energy added to the liquid by a pump (EP_n), minus the difference in energy grade between the end points of the path (DE_k) is equal to zero. For any loop or path k this may be expressed as

$$FE(Q)_k = \sum_{n \subset [k]}^{N_k} (HL_n + HM_n - EP_n) - DE_k = 0 \text{ for all } k \qquad (5)$$

where N_k = number of pipes associated with loop or path k; and $[k]$ = set of pipes associated with loop or path k.

Implicit Bound Constraints

The implicit bound constraints on the problem may include constraints on nodal pressures, pipe flow rates or velocities, and tank water levels. For each operational time interval, the pressure at junction node j may be bound between a maximum value $Pmax_{jt}$, and a minimum value $Pmin_{jt}$. This may be expressed as

$$Pmin_{jt} \leq P_{jt} \leq Pmax_{jt} \text{ for all } j,t \qquad (6)$$

Likewise, the flow rate Q_{kt} or velocity V_{kt} associated with any pipe k during time interval t may also be bound between maximum and minimum values expressed as

$$Qmin_{it} \leq Q_{it} \leq Qmax_{it} \text{ for all } i,t \qquad (7)$$

$$Vmin_{it} \leq V_{it} \leq Vmax_{it} \text{ for all } i,t \qquad (8)$$

In addition to constraints on pipe and node state variables it is also usually desirable to place restrictions on the water levels that may result from the implementation of the optimal control policy. Whereas the maximum allowable water level will normally be the top of the tank, the minimum allowable water level will normally be above the bottom of the tank in order to provide some residual storage for potential fire suppression activities. For each operational time interval t and tank l such constraints may be expressed as

$$Lmin_{lt} \leq L_{lt} \leq Lmax_{lt} \text{ for all } l,t \qquad (9)$$

In addition to these normal tank constraints, most optimal control policies are constructed to result in a set of tank trajectories that begin and end at specified target elevations. In most cases the beginning and ending levels will be the same. For such an operating strategy the following additional tank-water-level constraints would be required:

$$L_{lT} = L_{lo}, \text{ for all } l \qquad (10)$$

Explicit Bound Constraints

The final set of bound constraints consist of explicit bounds on the decision variables. In this case the decision variable for each pump station s for a particular time interval t will be restricted between a lower value of zero (corresponding to no pumps in operation) and an upper value equal to the maximum number of pump combinations available for that pump station (i.e. $II_{max} + 1$). Mathematically, this may be expressed as follows:

$$0 \leq X_{st} \leq II_{max} + 1 \text{ for all } s,t \qquad (11)$$

SOLUTION METHODOLOGY

In applying the proposed algorithm to a specific distribution system, the desired operating horizon (typically 24 hr) is divided into a discrete set of time intervals. A separate continuous decision variable for each time interval is then assigned to each pump station. To initiate the algorithm, a separate vector for each time interval is randomly generated or explicitly specified that contains the values of the decision variables for each pump station in the system. As a result, any potential solution will consist of a set of T vectors where T = number of time intervals that constitute the operating horizon. To ensure a feasible solution, the initially specified or generated set of decision vectors must satisfy the explicit bound constraints.

Similar to the original work of Chase and Ormsbee (1989, 1991) the proposed algorithm uses a disaggregated or dual-level solution methodology. This is accomplished by linking an optimization model with a network simulation model [i.e. KYPIPE2 (Wood 1991)]. Once an initial set of decision vectors is obtained from the optimization model, it is then passed to a network-simulation model for use in explicitly satisfying the implicit system constraints and for use in evaluating the implicit bound constraints. The values of the resulting state variables (i.e. flow rate, pressure, etc.) are then passed back to the optimization algorithm for use in quantifying the objective function and any violations in the implicit bound constraints. This information is then used to generate an improved set of decision vectors that automatically satisfies the explicit bound constraints and that seeks to minimize the objective function. Once generated, the improved set of decision vectors is then passed back to the simulation algorithm for subsequent evaluation. This process is then repeated until a specified level of convergence is obtained.

It should be emphasized that the time interval used in the control algorithm does not need to correspond to the time interval used in the associated network-simulation model. The only restriction is that the time interval of the control algorithm must be a multiple of the time interval of the network-simulation model. For example, while the network simulation model may use a 1-hr computational interval, the control algorithm may use a 12-hr control interval. The use of two separate time intervals thus allows for an improved accuracy with regard to the network computations while allowing for fewer decision variables to be considered by the control formulation.

Because of the nature of the prescribed continuous decision variable, the least-cost solution to the unconstrained objective function [i.e., (3)] is explicitly known. That is, the least-cost solution is one in which no pumps are operated. Obviously,
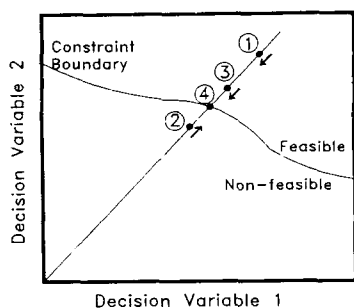
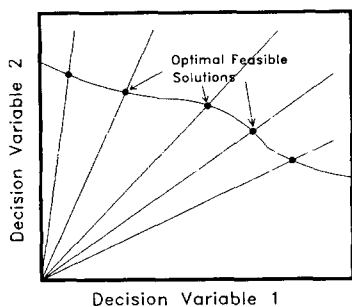**FIG. 1. Search Procedure in Two-Dimensional Space**
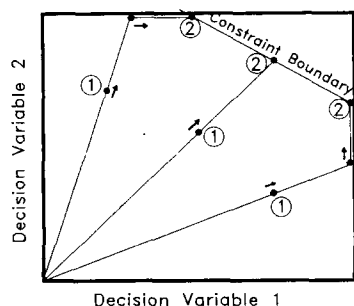

FIG. 2. Set of Feasible Solutions

FIG. 3. Identification of Feasible Solutions

such a solution will neither satisfy the system demands nor the associated implicit system constraints. As a result, the optimization problem reduces to one of finding the set of decision vectors that produce solutions on the implicit composite constraint boundaries that are as close to the origin of the T-dimensional solution space as possible. For example, consider a problem with two pump stations (i.e. two continuous decision variables) and a single control interval. For a given initial decision vector (point 1, Fig. 1) an improved vector may be obtained by contracting the scalar values in the vector toward the origin (point 2, Fig. 1). In the event that such a contraction results in an implicit bound constraint violation, then the vector that produced that constraint violation can be subsequently expanded until a feasible solution is obtained (point 3, Fig. 1). By continuing to bisect the resulting search direction, a set of decision variables can be obtained that will result in a solution that lies just outside (or on) the constraint boundary (point 4, Fig. 1).

It should be recognized that application of this methodology will only result in the best solution that lies on the search direction located between the given initial solution and the origin of the T-dimensional solution space. However, additional feasible solutions may be obtained by replicating the methodology using additional sets of initial decision vectors. Once the final set of feasible solutions is obtained, an "optimal" solution may be obtained by selecting the best solution from among the resulting feasible solutions (see Fig. 2). Indeed, not only is an "optimal" solution obtained, but all

resulting feasible solutions are available for examinations by the system operator. As a result, the operator is provided with an increased flexibility with regard to selection of alternative solutions that may not be optimal from a purely cost-savings objective but may provide a superior solution based on additonal more subjective operational considerations.

In the previous discussion it has been assumed that each search starts with an initial feasible solution. In the event that the initial solution is infeasible, a feasible solution can be obtained by expanding away from origin (i.e. by increasing the pump discharges) until a feasible solution is obtained. Unless the total system demand exceeds the aggregate pump capacity of the system, a feasible solution will eventually be found (point 2, Fig. 3).

APPLICATION

In an attempt to evaluate its feasibility, the proposed solution methodology was used to develop pump operating policies for the First High pressure zone of the Washington, D.C., distribution system for two different representative days in 1986. The costs associated with the historical operating policies were then compared with the costs associated with the optimal policies to assess the potential savings that could be obtained.

System Description

Water for Washington, D.C., is obtained from the Potomac River and then treated at two separate treatment plants, which are operated by the Washington Aqueduct Division of the Army Corps of Engineers. Pumping of the finished water is the responsibility of both the Washington Aqueduct Division and the Water Resources Management Administration of the District of Columbia Department of Environmental Services. The major pumping stations of the system are the Dalecarlia pump station (Corps of Engineers) and the Bryant Street pump station (Water Resources Management Administration). Ground elevations in the District of Columbia vary from under 7 ft to 420 ft above mean sea level. To provide an average water pressure of about 50 psi over this range in elevation, the city is divided into seven pressure zones, each comprising a certain range of ground elevations (see Fig. 4).

An optimal control algorithm for use with both the Second and Third High pressure zones has been developed and documented previously (Ormsbee et al. 1987). In that case, each pressure zone contained only one tank which permitted the use of a dynamic program formulation of the control problem (Ormsbee et al. 1989). In the current study, a more general control heuristic has been developed and applied to the First High pressure zone. The First High pressure zone provides service to that portion of Washington, D.C., just north and east of The Mall. The system also delivers water to portions of the city located south of the Potomac River. The average consumptive use for the D.C. First High system in 1986 was about 45 million gallons per day (MGD). A highly skeletonized schematic of the First High system is shown in Fig. 5.

The First High pressure zone is supplied from two pump stations: (1) The Dalecarlia pump station, operated by the Washington Aqueduct Division (WAD) of the U.S. Army Corps of Engineers; and (2) the Bryant Street pump station, operated by the Water Resources Management Administration (WRMA). The Dalecarlia pump station contains three 1,000 horsepower (HP) pumping units, each with a rated capacity of 35 MGD and a rated pump head of 145 ft; and the Bryant Street pump station contains three 800 HP pumps,
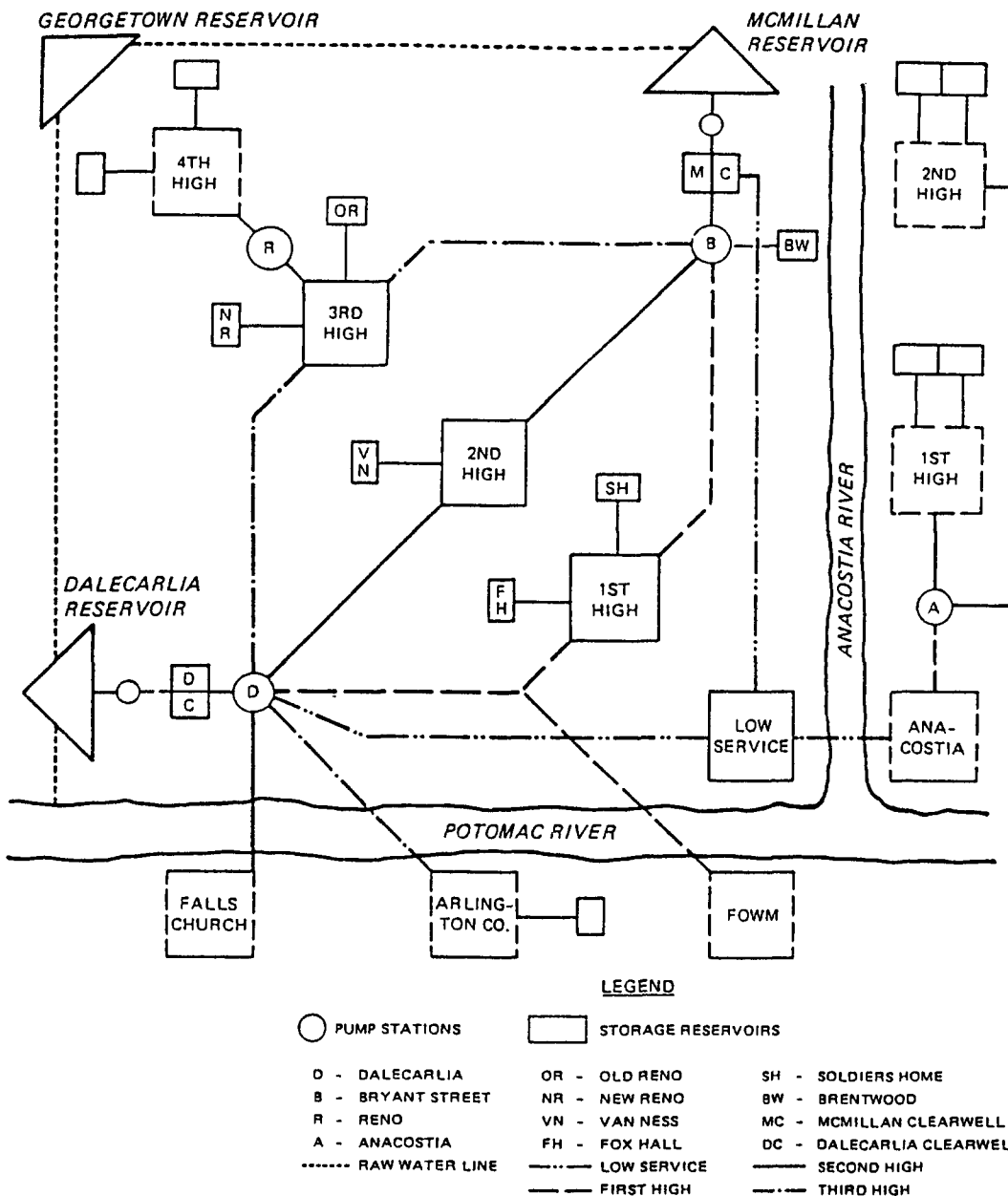
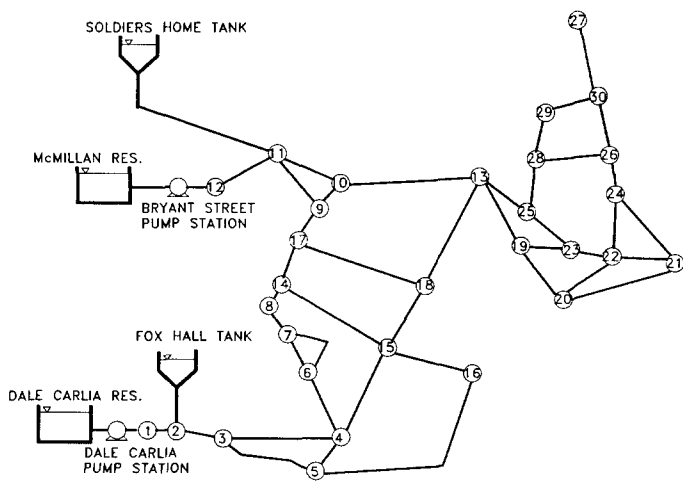FIG. 4. Washington, D.C., Pressure Zones

FIG. 5. Washington, D.C., First High Pressure Zone

each with a rated capacity of 35 MGD and a rated pump head of 110 ft.

In addition to the two pump stations, the First High pressure zone also contains two large concrete ground storage tanks. The Fox Hall tank has a capacity of 14.5 MG and is served primarily from the Dalecarlia pump station. The Soldiers Home tank has a capacity of 15 MG and is served primarily from the Bryant Street pump station. Both tanks have bottom elevations of 233 ft and overflow elevations of 250 ft.

Mathematical Model

Before applying the proposed control algorithm, a mathematical model of the First High distribution system was first developed. In order to decrease the computational requirements of the proposed application, a highly skeletonized model (i.e. 50 pipes) was developed. Prior to use of the model in the optimal control algorithm, the skeletonized model was first calibrated in order to ensure that the resulting model

adequately reflected both the steady-state and dynamic hydraulics of the actual system (Chase 1993).

Application Data

The proposed control algorithm was applied to the Washington, D.C., First High System for 2 days for which actual operating data was available. The first day, March 29, 1986, was a winter weekend day on which the electrical rate was a constant $0.0295/kW·h. The second day, June 11, 1986, was a summer weekday on which the electrical rate varied under a time-of-use schedule. Electrical costs for June 11 were $0.0295/ kW·h from midnight to 8:00 a.m., $0.0465/kW·h from 8:00 a.m. to 12:00 noon, $0.0624/kW·h from 12:00 noon to 8:00 p.m., and $0.0465/kW·h from 8:00 p.m. to midnight. Because of the fact that the demand charge for the Washington, D.C., system is calculated using the total peak consumption from all pump stations for all pressures zones and because of the fact that is assessed on an annual basis, its influence on the resulting daily policy for the First High System was not considered in this study.

While the total consumption for the First High System was 38 million gallons on March 29, 1986, it was 50,000,000 gal. on June 11, 1986. Hourly demand patterns for both days are shown in Figs. 6 and 7. For both days, the 24 hr operating horizon was subdivided into 24 one-hour time intervals. The hydraulics of the water-distribution-system network were then modeled using a 1-hr time interval, while control intervals
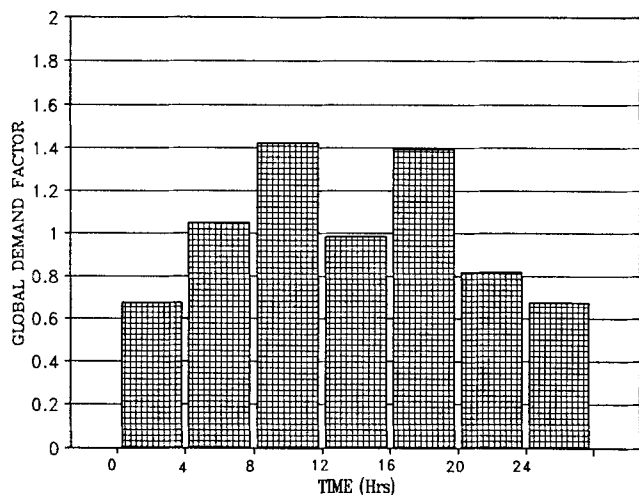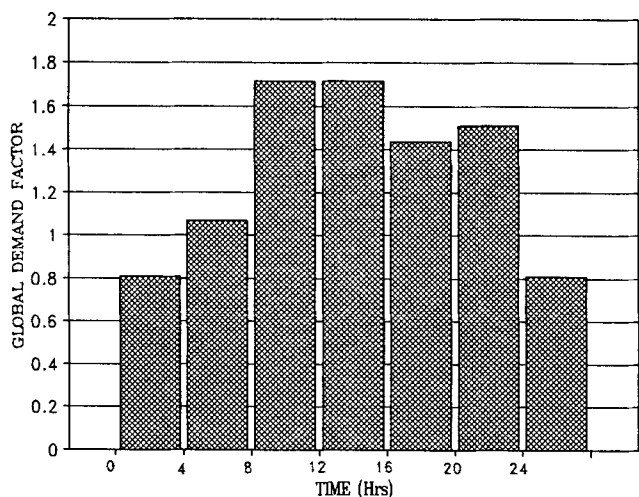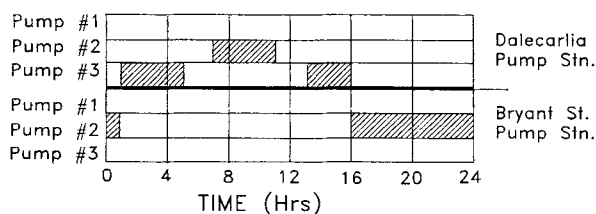

FIG. 8. Actual Pumping Policy for March 29, 1986
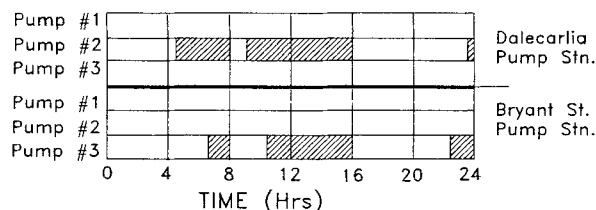

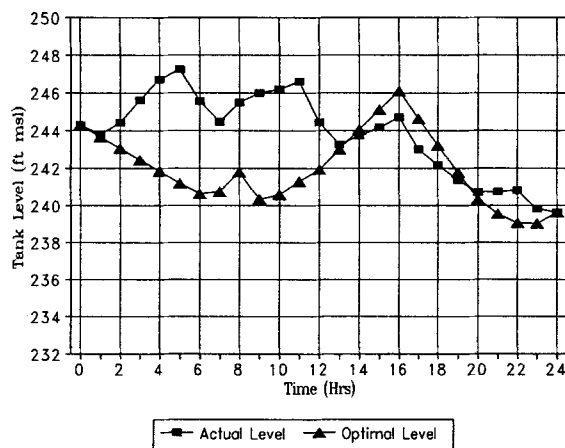FIG. 9. Optimal Pumping Policy for March 29, 1986

FIG. 10. Foxhall Tank Levels for March 29, 1986
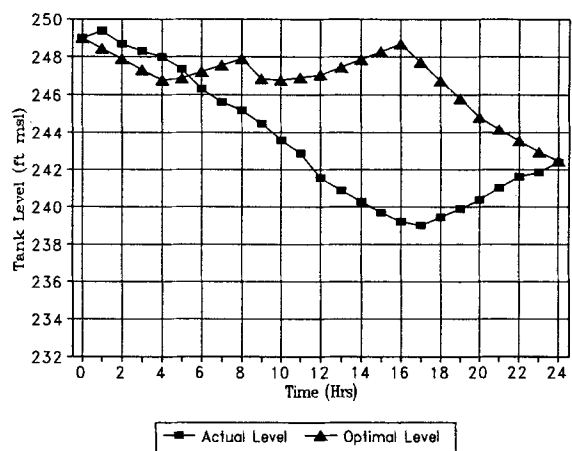

FIG. 11. Soldiers Home Tank Levels for March 29, 1986

FIG. 6. Temporal Demand Distribution for March 29, 1986

FIG. 7. Temporal Demand Distribution for June 11, 1986

TABLE 1. Optimal Solutions for March 29, 1986

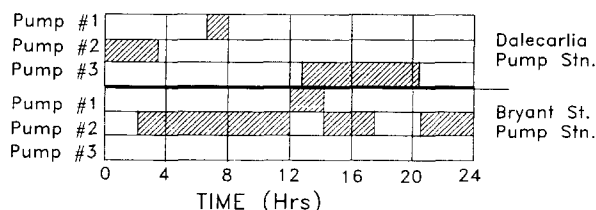| Number of control intervals (1)	Length of interval (hr) (2)	Cost of optimal solution ($) (3)	Number of solutions superior to actual policy (4)
2	12	385.00	1
3	8	361.00	15
4	6	362.00	16

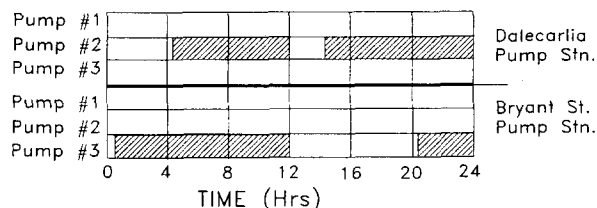FIG. 12. Actual Pumping Policy for June 11, 1986

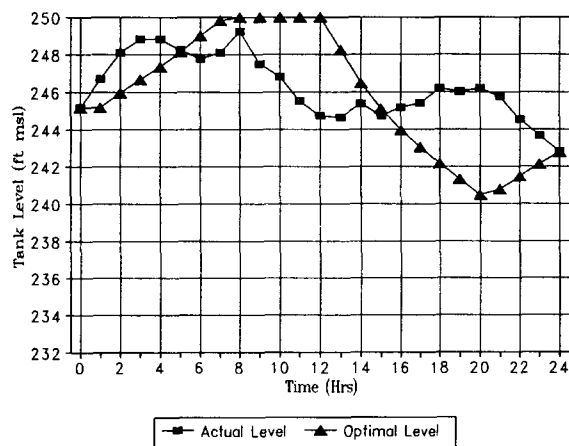
FIG. 13. Optimal Pumping Policy for June 11, 1986
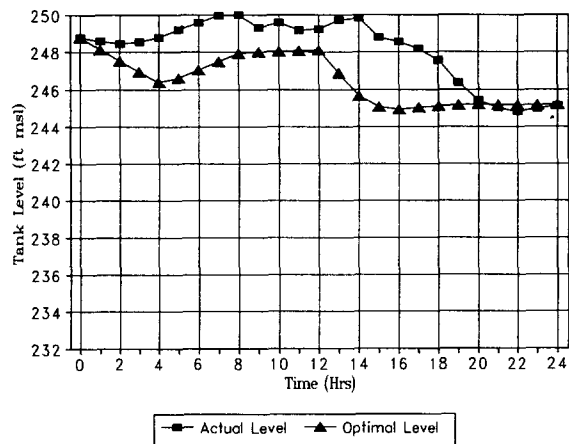

FIG. 14. Foxhall Tank Levels for June 11, 1986

FIG. 15. Soldiers Home Tank Levels for June 11, 1986

TABLE 2. Optimal Solutions for June 11, 1986

Number of control intervals (1)	Length of interval (hr) (2)	Cost of optimal solution ($) (3)	Number of solutions superior to actual policy (4)
2	12	857.00	15
3	8	882.00	6
4	6	868.00	6

ranging from 6 hr to 12 hr were investigated for use in developing the associated control policies.

System Constraints

For each application, the initial and final tank levels were restricted to coincide with the actual observed values for each day. For this study, the minimum tank levels were assumed to be 239.0 ft mean sea level (MSL). Such a constraint ensures a minimum of 6 ft of water in the storage tanks at all times and also corresponds to the historical operating policy. In each case, junction pressures were restricted to remain between 40 and 80 psi.

March 29 Application

For the first application, the control algorithm was applied to the Washington, D.C., First High pressure zone for March 29, 1986. The actual pump policy and resulting reservoir level trajectories associated with this day are shown in Figs. 8–11. The operating cost for this day was $387. In applying the control algorithm for March 29, 1986, three separate control intervals were investigated. These included 12 hr, 8 hr, and 6 hr. For each control interval considered, 100 random policies were first generated and then refined until a feasible set of policies were obtained. The optimal operating costs associated with each control interval are listed in Table 1 along with the total number of solutions that were superior to the existing policy. As shown in Table 1, the total operating cost associated with the optimal policy was found to be $361. This represents a 6.8% savings over the actual historic policy.

As can be seen from the table, the "global" optimal policy is the one associated with three 8-hr control intervals. The actual pump schedule associated with this policy is shown in Fig. 9, with the resulting reservoir level trajectories shown in Figs. 10 and 11. Somewhat surprisingly, the optimal tank trajectory for the Soldiers Home tank is frequently higher than the trajectory associated with the actual policy. This trend is somewhat compensated for, however, by the trajectory in the Foxhall tank. It should be remembered that the electric usage rate for this day is constant.

June 11 Application

In addition to March 29, 1986, the algorithm was also applied to June 11, 1986. This was done in order to evaluate the capabilities of the algorithm to handle a day with a variable time-of-use electrical-rate schedule. The operating cost for June 11, 1986 was $921. The associated pump policy and resulting tank-level trajectories are shown in Figs. 12–15. In applying the control algorithm for June 11, 1986, three separate control intervals were investigated. As before, these included time intervals of 12 hr, 8 hr, and 6 hr. For each control interval considered, 100 random policies were again generated and then refined until a feasible set of policies were obtained. The optimal operating costs associated with each control interval are listed in Table 2 along with the total number of solutions that were superior to the existing policy. As shown in Table 2, the total operating cost associated with the optimal policy was found to be $857. This represents a 6.9% savings over the actual historic policy.

In this case, the "global" optimal policy was the one associated with two 12-h control intervals. The actual pump schedule associated with this policy is shown in Fig. 13 with the resulting reservoir level trajectories shown in Figs. 14 and 15. Of particular interest is the fact that the optimal tank trajectories for both Foxhall and Soldiers Home reach their maximum level at 12 noon before beginning to decrease. In addition, both trajectories either stay the same or increase

back to their final levels from 8:00 p.m. to 12 midnight. This solution is consistent with the fact that the peak electric usage rate occurs from 12 noon to 8 p.m. In each case, the algorithm has generated a pump policy that minimizes the pumping that occurs during the period in which the electric usage rate is highest.

Run-Time Statistics

Of particular concern is the amount of time required to obtain an optimal pumping policy. Ideally the policy should be generated in as short a time span as is practical. The heuristic control model discussed here was executed on a 486 33 MHz IBM-compatible personal computer. Moreover, the program was compiled in standard FORTRAN to run in protected mode, i.e. 32-bit operation. In each case, the algorithm took approximately 40 min to obtain the resulting solution. This computation time could be reduced to 15 min while still yielding similar results, by using a 4-hr hydraulic computational time interval instead of 1 hr. Alternatively, the computational time could be reduced by reducing the total number of solutions to be considered. By way of comparison, application of a two-dimensional dynamic programming algorithm to each problem required in excess of 6 hr to obtain similar solutions for both March 29 and June 11 (Chase 1992).

SUMMARY AND CONCLUSIONS

Previous attempts to develop optimal control algorithms for water-distribution systems have typically focused on the development and use of implicit control formulations in which the problem is expressed in terms of an implicit state variable such as tank level or pump station discharge. Such formulations suffer from the requirement of a two-step optimization methodology in which the actual pump operating policies must be extracted from the solution of the implicit control problem (Ormsbee and Lansey 1994). Attempts to circumvent this problem by use of explicit formulations in which pump run times are treated as the decision variables are limited due to the number of decision variables that can effectively be considered (Chase and Ormsbee 1989, 1991). In the current study, this limitation is minimized by rank ordering different pump combinations and developing a single decision variable for each pump station for each control interval. Although further variable reduction could be accomplished by considering pump combinations that involve pumps from different pump stations, the writers have found that such a formulation results in solutions that are inferior to the proposed approach.

The proposed explicit formulation is solved by linkage of a nonlinear heuristic with a network simulation model. Although the rank offering of the available pump combinations does restrict the total number of pump policies that may be considered, the formulation is still able to provide for a range of feasible and cost-efficient solutions, as evidenced by the results of the example application. Finally, although the proposed heuristic does not possess the mathematical elegance of a more-sophisticated gradient-based optimization algorithm, its simplicity provides for a very efficient computational algorithm as well as the ability to provide for multiple feasible solutions. Linkage of the algorithm with a network-simulation model provides the added advantage of being able to explicitly incorporate changes to system topology and demand distribution as well as the ability to assess the impacts of the resulting policies on the pressures and flows throughout the system.

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APPENDIX. REFERENCES

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